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Mathematics II. (BSc)– Exam 2008

1. (8 p.) For which values of a and b will the system

$$\begin{bmatrix} 2 & 0 & -1 & 2 \\ 1 & 0 & 1 & 3 \\ 0 & 2 & -1 & 2 \\ 2 & 4 & 0 & a \end{bmatrix} \underline{x} = \begin{bmatrix} -1 \\ 2 \\ 1 \\ b \end{bmatrix}$$ have

a.) unique solution

b.) no solution

c.) infinitely many solutions. Give the solution set.

2. (8 p.) Find the relative extrem value(s) of $f(x,y) = 2xy + \frac{1}{x} - \frac{4}{y}$.

3. (9 p.) Determine whether the following series are absolutely convergent, conditionally convergent or divergent:

a.) $\sum_{n=2}^{\infty} \frac{(-1)^n}{\sqrt[n]{\ln 2}}$, b.) $\sum_{n=2}^{\infty} \frac{(-1)^n}{\sqrt{\ln n}}$, c.) $\sum_{n=2}^{\infty} \frac{(-1)^n}{n^2+8}$.

4. (7 p.) Find the arc length of the curve

$$\underline{r}(t) = 3t\underline{i} + 3t^2\underline{j} + 2t^3\underline{k}, \quad 0 \le t \le 1.$$

5. (10 p.) Determine the value of the following improper integrals:

a.) $\int_0^1 \frac{1-2x}{\sqrt{x(1-x)}} dx$, b.) $\int_0^{\infty} \frac{1}{1+e^x} dx$.

6. (8 p.) Let $A = \begin{bmatrix} 3 & -6 & 9 \\ 3 & -6 & 9 \\ 1 & -2 & 3 \end{bmatrix}$.

a.) $A^2 = ?$

b.) Show that $(I+A)^{-1} = I - A$.

c.) Find the eigenvalues of the matrix A.

17. (6 points) $\det \begin{pmatrix} 2 & 1 & 0 & -2 \\ 1 & 3 & 3 & -1 \\ 3 & 2 & 4 & -3 \\ 2 & -2 & 2 & 3 \end{pmatrix} = ?$

18. (6 points) $\det \begin{pmatrix} 2 & 0 & 0 & 4 \\ 0 & 6 & 0 & -2 \\ 4 & 0 & 2 & 0 \\ -2 & 6 & 0 & -2 \end{pmatrix} = ?$

19. (6 points) $\underline{\underline{A}} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$.

a.) $\det \underline{\underline{A}} = ?$ b.) $\det \underline{\underline{A}}^3 = ?$ c.) $\det 2\underline{\underline{A}}^{-1} = ?$ d.) $\det \left(\underline{\underline{A}} - \underline{\underline{A}}^T\right) = ?$

20. (16 points) Which statements are TRUE and which are FALSE? (Please, give reasons for your answer.)

a.) $x = 1, y = 1$ is the only solution of the system $x - y = 0$, $-2x + 2y = 0$.

b.) $\det \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{pmatrix} = 0$

c.) $\det \begin{pmatrix} 4 & 6 \\ -2 & 0 \end{pmatrix} = 2 \cdot \det \begin{pmatrix} 2 & 3 \\ -1 & 0 \end{pmatrix}$

d.) The system $\begin{pmatrix} 2 & -1 \\ -4 & 2 \end{pmatrix} \underline{x} = \underline{0}$ has only trivial solution.

e.) $\underline{\underline{A}} \cdot \underline{\underline{B}} = \underline{\underline{0}}$ only if $\underline{\underline{A}} = \underline{\underline{0}}$ or $\underline{\underline{B}} = \underline{\underline{0}}$.

f.) $\det\left(\underline{\underline{A}} + \underline{\underline{B}}\right) = \det\left(\underline{\underline{A}}\right) + \det\left(\underline{\underline{B}}\right)$.

g.) $\left(\underline{\underline{A}}^T\right)^{-1} = \left(\underline{\underline{A}}^{-1}\right)^T$.

h.) If $\underline{\underline{A}}^2 - 3\underline{\underline{A}} + \underline{\underline{E}} = \underline{\underline{0}}$, then $\underline{\underline{A}}^{-1} = 3\underline{\underline{E}} - \underline{\underline{A}}$