

Mathematical Statistics exam  
PhD Course

1. Let  $X_1, X_2, \dots, X_n$  a statistical sample from the Poisson distribution, where the parameter is  $\vartheta$ . Show that the sample mean,  $\bar{X}_n$  is sufficient statistics.

2. A matched sample of students in a school were enrolled in diving classes with different training techniques. Is there a difference?

Pairs	X Team A	Y Team B
1	37	35
2	39	46
3	32	24
4	21	34
5	20	28
6	9	12
7	14	9

3. Let  $X_1, X_2, \dots, X_n$  be statistical sample from  $N(\mu, \sigma)$ . Edit  $1 - \varepsilon$  level confidence interval for the parameter  $\sigma$ .

4. How do you interpret the next linear regression result:

Coefficients<sup>a</sup>

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.	Collinearity Statistics	
		B	Std. Error	Beta			Tolerance	VIF
1	(Constant)	-225,116	191,901		-1,173	,243		
	4-year resale value	,011	2,253	,002	,005	,996	,053	18,890
	Price in thousands	-,015	2,146	-,003	-,007	,994	,039	25,464
	Engine size	37,640	15,588	,529	2,415	,017	,134	7,474
	Horsepower	-,612	,342	-,477	-1,788	,077	,090	11,095
	Wheelbase	6,391	1,787	,685	3,576	,001	,175	5,718
	Width	-,375	3,120	-,018	-,120	,905	,298	3,352
	Length	-,437	1,077	-,081	-,406	,686	,163	6,149
	Curb weight	-69,476	29,602	-,553	-2,347	,021	,116	8,633
	Fuel capacity	-,157	3,697	-,008	-,043	,966	,184	5,437
	Fuel efficiency	-2,608	2,931	-,153	-,890	,376	,217	4,602

a. Dependent Variable: Sales in thousands

5. Describe the model of the principal components. How we calculate the principal directions? How we express the importance of the principal component? How we calculate the percent what can be explain with the redudated model ? What is the optimality of the PCA? Interpret the next table:

Total Variance Explained

Component	Initial Eigenvalues			Extraction Sums of Squared Loadings			Rotation Sums of Squared Loadings		
	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %
1	6,654	51,186	51,186	6,654	51,186	51,186	6,515	50,115	50,115
2	1,448	11,138	62,324	1,448	11,138	62,324	1,511	11,625	61,740
3	1,169	8,991	71,316	1,169	8,991	71,316	1,184	9,109	70,849
4	1,022	7,860	79,176	1,022	7,860	79,176	1,083	8,327	79,176
5	,867	6,669	85,845						
6	,546	4,196	90,041						
7	,471	3,625	93,666						
8	,306	2,357	96,023						
9	,272	2,096	98,118						
10	,125	,959	99,077						
11	,071	,547	99,624						
12	,040	,310	99,933						
13	,009	,067	100,000						

Extraction Method: Principal Component Analysis.