

SZÖRÁSAANALÍZIS

Adott egy X folytonos változó, ami normális eloszlású.

$$X \in N(\mu, \sigma)$$

Adottak ezen kívül az Y_1, Y_2, \dots, Y_k diszkrét változók (faktorok)

$$H_0 : X \text{ - re nincs hatással } Y_1$$

$$Q_{total} = Q_1 + Q_2 + \dots + Q_k + Q_{12} + Q_{13} + \dots + Q_{hiba}$$

A minta teljes szórásnégyzete $Q_{total} = \sum (x_i - \bar{x})^2$

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Az Y_1 magyarázta rész

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Az első két faktor interakciójához tartozó rész

Adott egy X folytonos változó, ami normális eloszlású.

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$$H_0 : X \text{ - re nincs hatással } Y_1$$

$$Q_{total} = Q_1 + Q_2 + \dots + Q_k + Q_{12} + Q_{13} + \dots + Q_{hiba}$$

A véletlen hiba okozta rész

Egyszerű csoportosítás

X A dolgozó fizetése

Y A dolgozó beosztása (tisztviselő, őrző-védő, menedzser)

$$H_0 : \text{A beosztás nincs hatással a fizetésre}$$

$$\{x_1^{(t)}, x_2^{(t)}, \dots, x_{n_t}^{(t)}\}, \{x_1^{(v)}, x_2^{(v)}, \dots, x_{n_v}^{(v)}\}, \{x_1^{(m)}, x_2^{(m)}, \dots, x_{n_m}^{(m)}\}$$

Egyszerű csoportosítás

Csoportátlagok:

$$\bar{x}^{(t)} = \frac{1}{n_t} \sum_{j=1}^{n_t} x_j^{(t)} \quad \bar{x}^{(v)} = \frac{1}{n_v} \sum_{j=1}^{n_v} x_j^{(v)} \quad \bar{x}^{(m)} = \frac{1}{n_m} \sum_{j=1}^{n_m} x_j^{(m)}$$

Négyzetösszegek:

$$Q_{total} = \sum_{j=1}^{n_t} (x_j^{(t)} - \bar{x})^2 + \sum_{j=1}^{n_v} (x_j^{(v)} - \bar{x})^2 + \sum_{j=1}^{n_m} (x_j^{(m)} - \bar{x})^2$$

$$Q_k = n_t (\bar{x}^{(t)} - \bar{x})^2 + n_v (\bar{x}^{(v)} - \bar{x})^2 + n_m (\bar{x}^{(m)} - \bar{x})^2$$

$$Q_b = \sum_{j=1}^{n_t} (x_j^{(t)} - \bar{x}^{(t)})^2 + \sum_{j=1}^{n_v} (x_j^{(v)} - \bar{x}^{(v)})^2 + \sum_{j=1}^{n_m} (x_j^{(m)} - \bar{x}^{(m)})^2$$

SZÖRÁSAANALÍZIS

Egyszerű csoportosítás

$$Q_{total} = Q_k + Q_b$$

H_0

 \Rightarrow

$$\frac{Q_k}{3-1}$$

$$\frac{Q_b}{n-3}$$

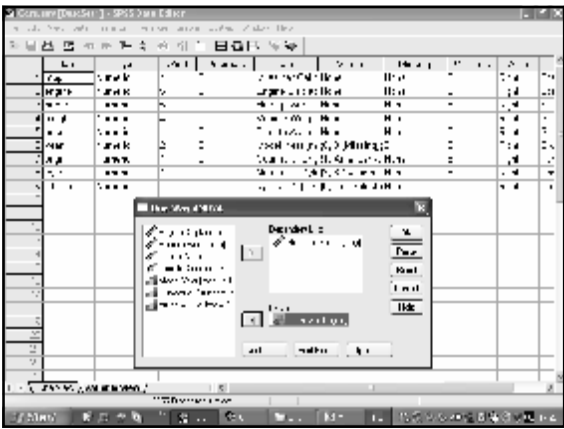
F-eloszlású (2, n-3)

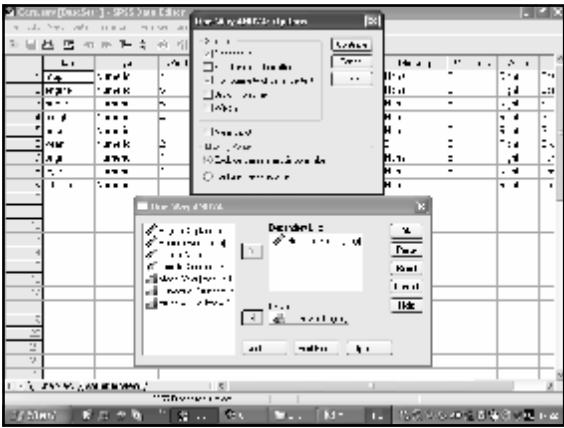
H_1

 \Rightarrow

$$\bar{x}^{(m)} - \bar{x}^{(t)} \pm t_{\varepsilon} \cdot \frac{Q_b}{n-3} \sqrt{\frac{n_m + n_t}{n_m \cdot n_t}}$$

Student (n-3)





Descriptives

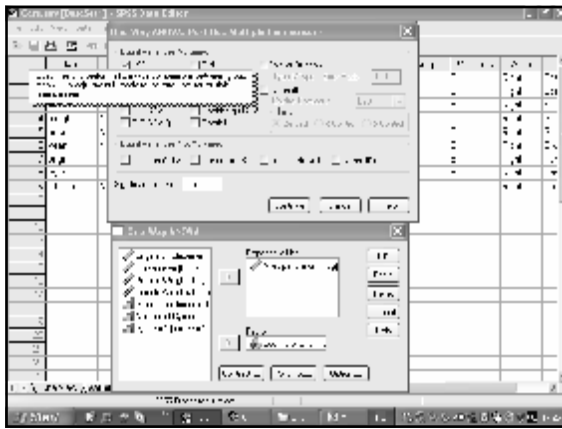
Miles per Gallon	N	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean		Minimum	Maximum
					Lower Bound	Upper Bound		
American	248	20,13	6,377	,405	19,33	20,93	10	39
European	70	27,89	6,724	,804	26,29	29,49	16	44
Japanese	79	30,45	6,090	,685	29,09	31,81	18	47
Total	397	23,55	7,792	,391	22,78	24,32	10	47

Test of Homogeneity of Variances

Miles per Gallon	Levene Statistic	df1	df2	Sig.
	,106	2	394	,900

ANOVA

Miles per Gallon	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	7884,957	2	3922,479	97,969	,000
Within Groups	16056,415	394	40,752		
Total	24041,372	396			



Report

Miles per Gallon	Country of Origin	Mean	N	Std. Deviation
	American	20,13	248	6,377
	European	27,89	70	6,724
	Japanese	30,45	79	6,090
Total		23,55	397	7,792

Multiple Comparisons

Dependent Variable: Miles per Gallon

LSD

(I) Country of Origin	(J) Country of Origin	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
American	European	-7,763*	,864	,000	-9,46	-6,06
	Japanese	-10,322*	,825	,000	-11,94	-8,70
European	American	7,763*	,864	,000	6,06	9,46
	Japanese	-2,559 [#]	1,048	,015	-4,62	-,50
Japanese	American	10,322*	,825	,000	8,70	11,94
	European	2,559 [#]	1,048	,015	-,50	4,62

*. The mean difference is significant at the .05 level.
[#]. The mean difference is significant at the .05 level.

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I.

Y függőváltozó

X_1, X_2, \dots, X_p független változók

$Y \approx f(X_1, X_2, \dots, X_p)$ becslés $f \in F$

$$E(Y - f(X_1, X_2, \dots, X_p))^2 = \min_{f \in F} E(Y - f(X_1, X_2, \dots, X_p))^2$$

A legkisebb négyzetek módszere

$$h(a, b, c, \dots) = \sum_{i=1}^n (Y_i - f(X_{1i}, X_{2i}, \dots, X_{pi}; a, b, c, \dots))^2 \rightarrow \min_{a, b, c, \dots}$$

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II.

• Lineáris regresszió $f(X) = B_0 + B_1 X$

• Többváltozós lineáris regresszió

$$f(X_1, X_2, \dots, X_p) = B_0 + B_1 X_1 + B_2 X_2 + \dots + B_p X_p$$

• Polinomiális regresszió

$$f(X_1, X_2, \dots, X_p) = B_0 + B_1 X + B_2 X^2 + \dots + B_p X^p$$

$$X_1 = X, X_2 = X^2, \dots, X_p = X^p$$

• Kétparaméteres (lineárisra visszavezethető) regresszió

$$\text{pl. } Y = f(X) = B_0 \cdot e^{B_1 X} \Rightarrow \ln Y = B_1 X + \ln B_0$$

Kétparaméteres (lineárisra visszavezethető) regresszió



$y = b_0 + b_1 x + b_2 x^2$	quadratic	$y = \exp\left(b_0 + \frac{b_1}{x}\right)$	S
$y = b_0 \cdot b_1^x$	compound	$y = b_0 + \exp(b_1 \cdot x)$	exponential
$y = \exp(b_0 + b_1 \cdot x)$	growth	$y = b_0 + \frac{b_1}{x}$	inverse
$y = b_0 + b_1 \cdot \ln x$	logarithmic	$y = b_0 + x^{b_1}$	power
$y = b_0 + b_1 \cdot x + b_2 \cdot x^2 + b_3 \cdot x^3$	cubic	$y = \frac{1}{1/u + b_0 + b_1^x}$	logistic

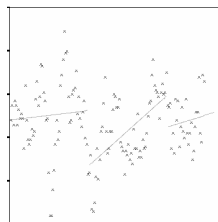
REGRESSZIÓANALÍZIS	• Nemlineáris regresszió	
	$f(X) = B_1 + B_2 \exp(B_3 X)$	aszimptotikus I.
	$f(X) = B_1 - B_2 \cdot (B_3)^X$	aszimptotikus II.
	$f(X) = (B_1 + B_2 X)^{1/B_3}$	sűrűség
	$f(X) = B_1 \cdot (1 - B_3 \cdot \exp(B_2 X^2))$	Gauss
	$f(X) = B_1 \cdot \exp(-B_2 \exp(-B_3 X^2))$	Gompertz
	$f(X) = B_1 \cdot \exp(-B_2/(X + B_3))$	Johnson-Schumacher
III.		

REGRESSZIÓANALÍZIS	• Nemlineáris regresszió	
	$f(X) = (B_1 + B_3 X)^{B_2}$	log-módosított
	$f(X) = B_1 - \ln(1 + B_2 \exp(-B_3 X))$	log-logisztikus
	$f(X) = B_1 + B_2 \exp(-B_3 X)$	Metcherlich
	$f(X) = B_1 \cdot X / (X + B_2)$	Michaelis Menten
	$f(X) = (B_1 B_2 + B_3 X^{B_1}) / (B_2 + X^{B_1})$	Morgan-Merczer-Florin
	$f(X) = B_1 / (1 + B_2 \exp(-B_3 X + B_4 X^2 + B_5 X^3))$	Peal-Reed
IV.		

REGRESSZIÓANALÍZIS	• Nemlineáris regresszió	
	$f(X) = (B_1 + B_2 X + B_3 X^2 + B_4 X^3) / B_5 X^3$	köbök aránya
	$f(X) = (B_1 + B_2 X + B_3 X^2) / B_4 X^2$	négyzetek aránya
	$f(X) = B_1 / ((1 + B_3 \cdot \exp(B_2 X))^{1/B_4})$	Richards
	$f(X) = B_1 / (1 + B_3 \cdot \exp(B_2 X))$	Verhulst
	$f(X) = (B_1^{(1-B_4)} \cdot B_2 \exp(-B_3 X))^{1/(1-B_4)}$	Von Bertalanffy
	$f(X) = B_1 - B_2 \exp(-B_3 X^{B_4})$	Weibull
$f(X) = 1 / (B_1 + B_2 X + B_3 X^2)$	Yield sűrűség	
V.		

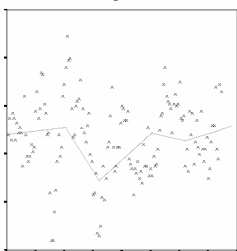
• Szakaszonkénti lineáris regresszió

$$f(X) = \begin{cases} A_1X + B_1 & X \in [t_1, t_2) \\ A_2X + B_2 & X \in [t_2, t_3) \\ \vdots & \vdots \\ A_cX + B_c & X \in [t_c, t_{c+1}) \end{cases}$$



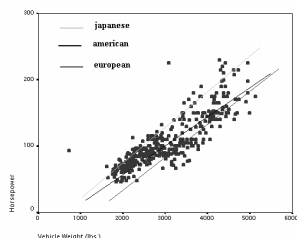
• Poligoniális regresszió

$$f(X) = \begin{cases} A_1X + B_1 & X \in [t_1, t_2) \\ A_2X + B_2 & X \in [t_2, t_3) \\ \vdots & \vdots \\ A_cX + B_c & X \in [t_c, t_{c+1}) \end{cases}, \quad A_i t_{i+1} + B_i = A_{i+1} t_{i+1} + B_{i+1}$$



• Többváltozós lineáris regresszió kategória-változóval

$$f(X) = \begin{cases} B_0 + B_1 X & \text{ha } K = c, \\ (B_0 + B_2) + (B_1 + B_{c+1})X & \text{ha } K = c-1, \\ (B_0 + B_3) + (B_1 + B_{c+2})X & \text{ha } K = c-2, \\ \vdots & \vdots \\ (B_0 + B_c) + (B_1 + B_{2c-1})X & \text{ha } K = c-1 \end{cases}$$



• Logisztikus regresszió

Y dichotóm $Y = \begin{cases} 1, & \text{ha az A esemény bekövetkezik} \\ 0, & \text{ha az A esemény nem következik be} \end{cases}$

A

- A választó fog szavazni
- A páciensnek szívinfarktusa lesz
- Az üzletet meg fogják kötni

X_1, X_2, \dots, X_p **ordinális szintű független változók**

- eddig hányszor ment el, kor, iskola, jövedelem
- napi cigi, napi pohár, kor, stressz
- ár, mennyiség, piaci forgalom, raktárkészlet

• Logisztikus regresszió

$$P(Y=1) = P(A) \approx \frac{1}{1 + e^{-Z}}$$

$$Z = B_0 + B_1 X_1 + B_2 X_2 + \dots + B_p X_p$$

$$\text{ODDS} = \frac{P(A)}{1 - P(A)} \approx e^Z \Rightarrow$$

$$\log(\text{ODDS}) = Z = B_0 + B_1 X_1 + B_2 X_2 + \dots + B_p X_p$$

• Logisztikus regresszió

A legnagyobb valószínűség elve

$$L(\epsilon_1, \epsilon_2, \dots, \epsilon_n) = P(Y_1 = \epsilon_1, Y_2 = \epsilon_2, \dots, Y_n = \epsilon_n) =$$

$$= P(Y_1 = \epsilon_1) P(Y_2 = \epsilon_2) \dots P(Y_n = \epsilon_n) \approx$$

$$\approx \frac{1}{1 + e^{-Z_1}} \cdot \frac{1}{1 + e^{-Z_2}} \cdot \dots \cdot \frac{1}{1 + e^{-Z_n}}$$

$$\ln L(\epsilon_1, \epsilon_2, \dots, \epsilon_n) = \sum \ln \left(\frac{1}{1 + \exp(B_0 + B_1 X_1 + B_2 X_2 + \dots + B_p X_p)} \right)$$

• Lineáris regresszió

A lineáris kapcsolat kitétetett:

- (1) a legegyszerűbb és leggyakoribb.
- (2) két dimenziós normális eloszlás esetén a kapcsolat nem is lehet más (vagy lineáris vagy egyáltalán nincs).

$$B_1 = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^n (x_i - \bar{x})^2}$$

• Lineáris regresszió

A teljes négyzetösszeg

$$SSTO = Q = \sum_{i=1}^n (y_i - \bar{y})^2$$

A maradékösszeg

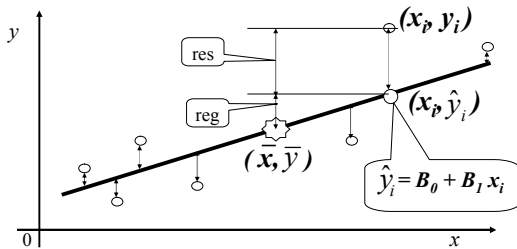
$$SSE = Q_{res} = \sum_{i=1}^n e_i^2 = \sum_{i=1}^n (y_i - \tilde{y}_i)^2 = \sum_{i=1}^n (y_i - B_0 - B_1 x_i)^2$$

A regressziós összeg

$$SSR = Q_{reg} = \sum_{i=1}^n (\tilde{y}_i - \bar{y})^2$$

A lineáris regresszió

$$Q = Q_{res} + Q_{reg}$$



A lineáris regresszió

A teljes négyzetösszeg felbontása:

$$Q = Q_{res} + Q_{reg}$$

f_{reg} szabadsági foka $n-2$, mert n tagú az összeg, de ezek között két összefüggés van.

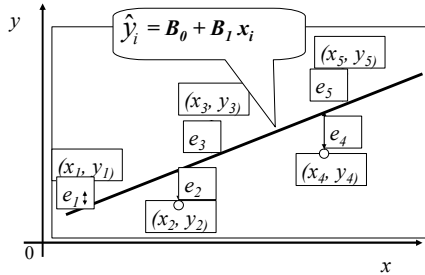
f_{res} szabadsági foka mindössze 1, mert az átlag konstans

Ha nincs lineáris regresszió, a varianciák hányadosa $(1, n-2)$ szabadsági fokú F eloszlást követ.

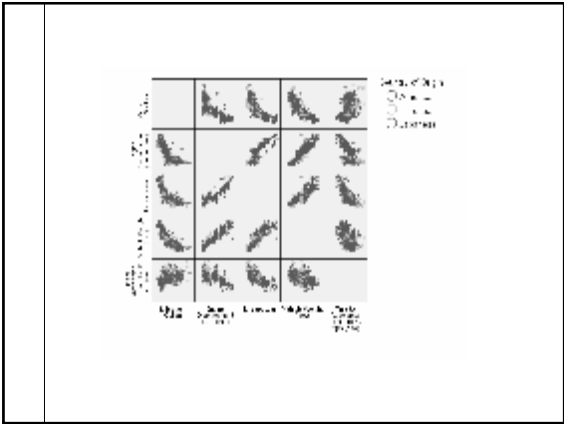
$$F = \frac{S_{reg}^2}{S_{res}^2} = \frac{f_{reg}}{f_{res}} = \frac{Q_{reg}(n-2)}{Q_{res}}$$

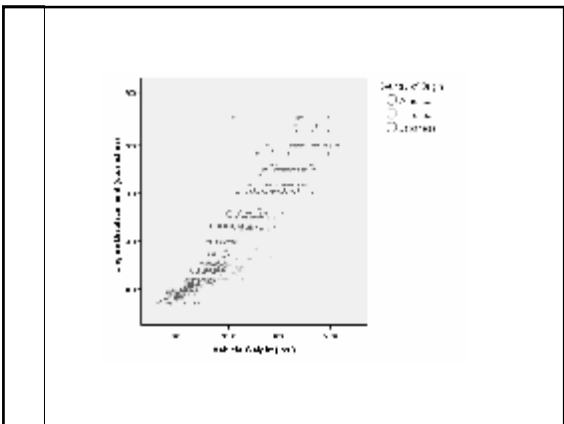
A lineáris regresszió

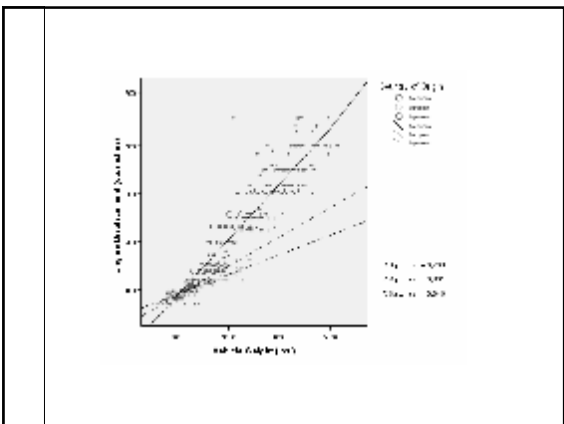
A legkisebb négyzetek módszere alapelve:

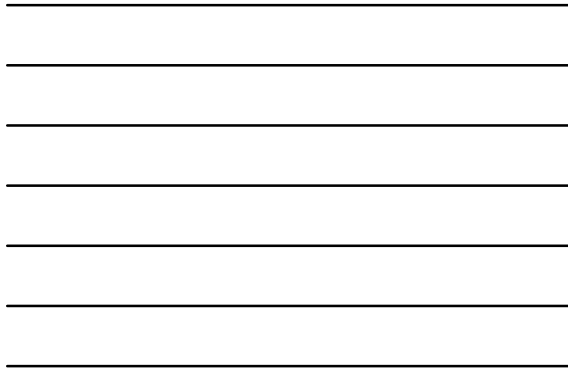
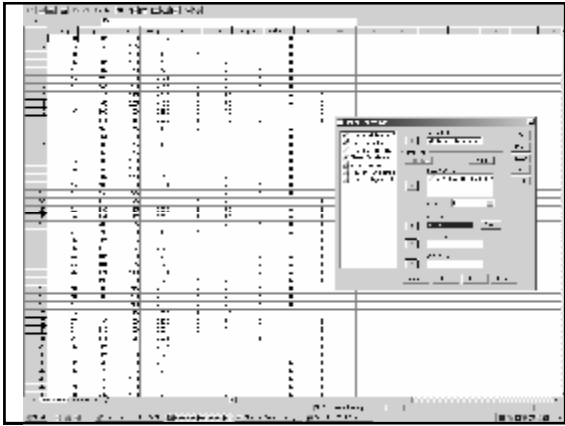












Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.924 ^a	.846	.843	38.868

a. Predictors: (Constant), Vehicle Weight (lbs.)

ANOVA^a

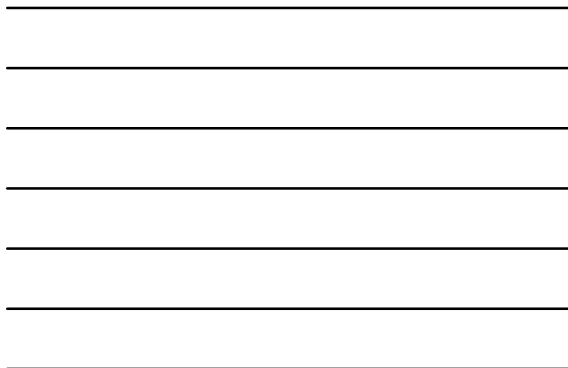
Model	Sum of Squares	df	Mean Square	F	Sig.
1	2079737	1	2079737.024	1376.808	.000 ^b
	Residual	251	1510.552		
	Total	252			

a. Predictors: (Constant), Vehicle Weight (lbs.)
 b. Dependent Variable: Engine Displacement (cu. inches)
 c. Selecting only cases for which Country of Origin = American

Coefficients^a

Model	Unstandardized Coefficients		Standardized Coefficients		Sig.	
	B	Std. Error	Beta	t		
1	(Constant)	-140.192	107.706		13.058	.000
	Vehicle Weight (lbs.)	.115	.003	.920	37.105	.000

a. Dependent Variable: Engine Displacement (cu. inches)
 b. Selecting only cases for which Country of Origin = American



Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.895 ^a	.801	.798	10.045

a. Predictors: (Constant), Vehicle Weight (lbs.)

ANOVA^a

Model	Sum of Squares	df	Mean Square	F	Sig.
1	8872.390	1	8872.390	286.154	.000 ^b
	Residual	71	100.898		
	Total	72			

a. Predictors: (Constant), Vehicle Weight (lbs.)
 b. Dependent Variable: Engine Displacement (cu. inches)
 c. Selecting only cases for which Country of Origin = European

Coefficients^a

Model	Unstandardized Coefficients		Standardized Coefficients		Sig.	
	B	Std. Error	Beta	t		
1	(Constant)	10.275	5.999		1.718	.090
	Vehicle Weight (lbs.)	.041	.002	.895	16.916	.000

a. Dependent Variable: Engine Displacement (cu. inches)
 b. Selecting only cases for which Country of Origin = European

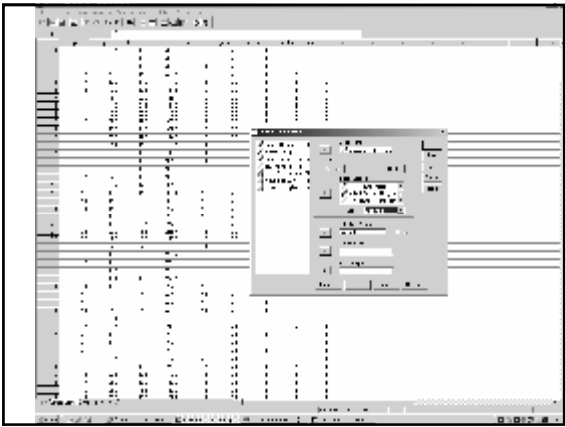


ANOVA ^a						
Model	Sum of Squares	df	Mean Square	F	Sig.	
1	Regression	7482,899	4	1870,725	177,730	,000 ^b
	Residual	2515,611	239	10,526		
	Total	9998,529	243			

a. Predictors: (Constant), Engine Displacement (cu. inches), Time to Accelerate from 0 to 60 mph (sec), Horsepower, Vehicle Weight (lbs.)
b. Dependent Variable: Miles per Gallon
c. Selecting only cases for which Country of Origin = American

Coefficients ^{a,b}						
Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error			
1	(Constant)	48,620	2,489		18,661	,000
	Horsepower	-.019	,015	-.117	-1,269	,209
	Vehicle Weight (lbs.)	-.003	,001	-.342	-3,642	,000
	Time to Accelerate from 0 to 60 mph (sec)	-.429	,129	-.183	-3,315	,001
	Engine Displacement (cu. inches)	-.034	,007	-.529	-4,802	,000

a. Dependent Variable: Miles per Gallon
b. Selecting only cases for which Country of Origin = American



Variables Entered/Removed ^a			
Model	Variables Entered	Variables Removed	Method
1	Vehicle Weight (lbs.)		Stepwise (Criteria: Probability >= .050, Probable y-of, F-to-enter <= .050, Probable y-of, F-to-remove <= .100)
2	Engine Displacement (cu. inches)		Stepwise (Criteria: Probability >= .050, Probable y-of, F-to-enter <= .050, Probable y-of, F-to-remove <= .100)
3	Time to Accelerate from 0 to 60 mph (sec)		Stepwise (Criteria: Probability >= .050, Probable y-of, F-to-enter <= .050, Probable y-of, F-to-remove <= .100)

a. Dependent Variable: Miles per Gallon
b. Models are based only on cases for which Country of Origin = American

Model Summary				
Model	Country of Origin = American (Selected)	R	Adjusted R Square	Std. Error of the Estimate
1	.851 ^a	.713	.717	3.442
2	.858 ^b	.736	.734	3.308
3	.865 ^c	.747	.744	3.248

a. Predictors: (Constant), Vehicle Weight (lbs.)
b. Predictors: (Constant), Vehicle Weight (lbs.), Engine Displacement (cu. inches)
c. Predictors: (Constant), Vehicle Weight (lbs.), Engine Displacement (cu. inches), Time to Accelerate from 0 to 60 mph (sec)

ANOVA ^d						
Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	7133.810	1	7133.810	601.987	.000 ^e
	Residual	2866.919	242	11.847		
	Total	9999.529	243			
2	Regression	7380.862	2	3690.431	336.298	.000 ^e
	Residual	2637.538	241	10.944		
	Total	9999.529	243			
3	Regression	7466.203	3	2488.734	235.869	.000 ^e
	Residual	2532.326	240	10.551		
	Total	9999.529	243			

d. Predictors: (Constant), Vehicle Weight (lbs.)
e. Predictors: (Constant), Vehicle Weight (lbs.), Engine Displacement (cu. inches)
f. Predictors: (Constant), Vehicle Weight (lbs.), Engine Displacement (cu. inches), Time to Accelerate from 0 to 60 mph (sec)
g. Dependent Variable: Miles per Gallon
h. Selecting only cases for which Country of Origin = American

Coefficients ^{a,b}						
Model		Unstandardized Coefficients		Standardized Coefficients		Sig.
		B	Std. Error	Beta	t	
1	(Constant)	43.104	.964		44.715	.000
	Vehicle Weight (lbs.)	-.007	.000	-.845	-24.526	.000
2	(Constant)	39.642	1.196		33.148	.000
	Vehicle Weight (lbs.)	-.004	.001	-.490	-5.811	.000
	Engine Displacement (cu. inches)	-.025	.005	-.386	-4.578	.000
3	(Constant)	44.713	1.989		22.476	.000
	Vehicle Weight (lbs.)	-.003	.001	-.377	-4.176	.000
	Engine Displacement (cu. inches)	-.038	.007	-.540	-5.626	.000
	Time to Accelerate from 0 to 60 mph (sec)	-.336	.107	-.143	-3.158	.002

a. Dependent Variable: Miles per Gallon
b. Selecting only cases for which Country of Origin = American

Excluded Variables ^c						
Model		Beta In	t	Sig.	Partial Correlation	Collinearity Statistics Tolerance
1	Horsepower	-.140 ^d	-2.543	.006	-.143	.301
	Time to Accelerate from 0 to 60 mph (sec)	.009 ^d	.226	.822	.015	.794
	Engine Displacement (cu. inches)	-.386 ^d	-4.578	.000	-.283	.154
2	Horsepower	.059 ^d	.762	.453	.049	.180
	Time to Accelerate from 0 to 60 mph (sec)	-.143 ^d	-3.158	.002	-.200	.513
	Horsepower	-.117 ^d	-1.259	.209	-.081	.122

a. Predictors in the Model: (Constant), Vehicle Weight (lbs.)
b. Predictors in the Model: (Constant), Vehicle Weight (lbs.), Engine Displacement (cu. inches)
c. Predictors in the Model: (Constant), Vehicle Weight (lbs.), Engine Displacement (cu. inches), Time to Accelerate from 0 to 60 mph (sec)
d. Dependent Variable: Miles per Gallon

