

Name: \_\_\_\_\_

*First part*

*Second Part*

1.	2.	3.	4.	$\sum$	5.	6.	7.	8.	$\sum$
4/	6/	6/	9/	25 ___	6/	7/	6/	6/	25 ___

**Mathematics I. (BSc)– 1st Midterm Test  
21st of October, 2010.**

**1. Examples of the First Part**

(You need reach at least 8 points to pass this part.)

2. (4 p.) Find the values of  $a$  and  $b$  that makes the function  $f$  be continuous.

$$f(x) = \begin{cases} \sin\left(x + \frac{\pi}{4}\right) + 2x & , \text{ if } x \leq 0 \\ \frac{x^2 - 3x}{\sqrt{x}} + a & , \text{ if } 0 < x \leq 1 \\ \frac{bx^2 + 3}{x+1} & , \text{ if } x > 1 \end{cases}$$

3. (6 p.) Find the limits:

a.)  $\lim_{x \rightarrow 4} \frac{x^2 - x - 12}{\sqrt{x^2 + 9} - 5},$       b.)  $\lim_{x \rightarrow 0^+} \frac{x^2 - 2x}{1 - \cos \sqrt{x}},$       c.)  $\lim_{x \rightarrow \infty} \frac{e^{2x} - 5e^{3x} - 1}{3e^{2x} + 8e^{3x} + 7}.$

4. (6 p.) Let

$$f(x) = -\frac{\pi}{2} + \frac{1}{3} \tan^{-1} \ln x.$$

a.) Find the domain and the range of  $f$ .

b.) Show that  $f$  has the inverse function. Give the inverse function of it.

c.)  $f'(x) = ?$ ,  $(f^{-1}(x))' = ?$

5. (9 p.) Determine the derivatives of the following functions:

a.)  $f(x) = (1 + x^2)^{x+1}$ ,      b.)  $g(x) = \frac{5x^2 - \sqrt{\ln x}}{\sin(x^3 + 2x)}$ ,

c.)  $h(x) = 4^{2x+1} + \log_2(5x + 3) + \sin^{-1}(\sqrt{1+x})$ .

### Examples of the Second Part

(You need reach at least 8 points to pass this part.)

6. (6 p.) Find parametric equation for the line through  $P(2, -1, 3)$  that is parallel

to the planes  $T : x - y + z = 0$  and  $S : 2x + y - z = 1$ .

7. (7 p.) a.) Calculate and give the result in algebraic form:  $z = \sqrt[3]{-1}(1+i)^4$ .

b.) Solve the equation  $z^2 = \bar{z}$ .

8. (6 p.) Find the limit of each convergent sequence:

a.)  $\lim_{n \rightarrow \infty} \sqrt[n]{\frac{4n^2 + 3}{n^2 + 4}}$ ,      b.)  $\lim_{n \rightarrow \infty} \sqrt[n]{2^{2n+3} \left(\frac{n+1}{n-4}\right)^{2n^2}}$ ,      c.)  $\lim_{n \rightarrow \infty} \frac{3^n + n^2 2^n}{2^{3n+1} + 5}$ .

9. (6 p.) Consider the vectors  $\mathbf{a} = (2 \ -1 \ 1)$  and  $\mathbf{b} = (1 \ 2 \ 3)$ .

a.) Find  $\mathbf{a} \times \mathbf{b}$ .

b.) Determine the angle between  $\mathbf{a}$  and  $\mathbf{b}$ .

c.) Compute the signed length of the perpendicular projection of  $\mathbf{b}$  on  $\mathbf{a}$ .