Name:

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Mathematics II. (BSc)– Extra2 Test
21st of May, 2014.
90 minutes
(You need reach at least 15 points to pass.)

1. (6 p.) Solve the matrix equation:

$$\begin{pmatrix} 0 & 1 \\ 2 & 3 \end{pmatrix} \underline{\underline{X}} = \begin{pmatrix} -7 & 2 & -13 & -10 \\ -8 & 3 & -8 & -7 \end{pmatrix}.$$

2. (7 p.) Construate an $\underline{e}_1, \underline{e}_2, \underline{e}_3$ orthonormal basis with Gram-Schmidt method, where $\underline{e}_1$ parallel with $\underline{a}_1$.

$$\underline{a}_1 = (0,1,2), \quad \underline{a}_2 = (2,0,-2), \quad \underline{a}_3 = (3,3,0).$$

3. (5 p.)

$$\text{rank} \begin{pmatrix} t & 0 & t^2 - 2 \\ 0 & 0 & 1 \\ -1 & t & t - 1 \end{pmatrix} = ?$$

4. (7 p.)

a.) $\mathcal{L}(t \sinh 2t) = ?,$ b.) $\mathcal{L}^{-1}\left(\dfrac{3s}{s^2 + s - 2}\right) = ?,$

c.) Solve the next differential equation using Laplace transform:
$$y' - 3y = 9x, \quad y(0) = 1.$$

5. (6 p.) Find the value of the integral if it exist:

$$\int_{2}^{\infty} \frac{1}{x^2 + x - 2}\,dx.$$

6. (6 p.) Find Taylor series at $x_o = 2$ for the function

$$f(x) = \frac{1}{x + 5}$$

and give the domain of the convergence.

7. (6 p.) Let the function

$$f(x,y) = \frac{e^{4x+y^2}}{3 - y}$$

is an equation of a surface.
   a.) Find the gradient of the function at $P_0(0,2)$?
   b.) Give the equation of the tangent plane at $P_0(0,2)$.
   c.) Calculate the directional derivative of $f(x,y)$ at $P_0(0,2)$ in the direction
      $v = (3,-4)$.

8. (7 p.) Solve the following differential equation:

$$y'' + 8y' + 12y = 3x, \quad y(0) = 1, \quad y'(0) = -1.$$