

Name: _____

1.	2.	3.	4.	5.	6.	7.	8.	Σ
6/	6/	6/	6/	7/	6/	7/	6/	50__

Mathematics II. (BSc)– Exam 1.
Jan 7 , 2016.

You need reach at least 20 points to pass.

1. (6 p.) Given the matrix

$$\underline{A} = \begin{pmatrix} 3 & 2 & 1 \\ 4 & -1 & 1 \\ 7 & 1 & 2 \end{pmatrix},$$

- Give the rank of the matrix
- What is the determinant of the matrix?
- What is the determinant of \underline{A}^{10} ?

2. (6 p.) For which values of a and b will the system

$$\begin{aligned} 2x_1 & & - x_3 & + 2x_4 & = & -1 \\ & x_2 & + x_3 & + 3x_4 & = & 2 \\ -4x_1 & & +2x_3 & + ax_4 & = & b \\ 2x_1 & & -3x_3 & + 3x_4 & = & -1 \end{aligned}$$

have

- unique solution,
 - infinitely solutions,
 - no solution.
3. (6 p.) Solve the next differential equation using Laplace transform:

$$y' + y = \sin 3t, \quad y(0) = 0.$$

4. (6 p.) Solve the following differential equation:

$$y'' + y' - 30y = e^{5x}, \quad y(0) = 1, \quad y'(0) = 0.$$

5. (7 p.) a.) Find the values of the integrals:

$$\text{a.) } \int_3^{\infty} \frac{1}{x\sqrt{x+1}} dx, \quad \text{b.) } \iint_{x^2+y^2 \leq 9} e^{-(x^2+y^2)} dx dy.$$

6. (6 p.) Given the function

$$f(x, y) = e^{y^2-x-1}(2x+1)^5$$

and a point $P_o(-1, 0)$.

a.) Find the derivative of f at P_o in the direction of $\underline{v} = -3\mathbf{i} + 4\mathbf{j}$.

b.) Find an equation for the tangent plane at the point P_o on the given surface.

7. (7 p.) Find the absolute maximum and minimum values of

$$f(x, y) = x^2 + y^2 + xy$$

in the unit square $0 \leq x \leq 1$, $0 \leq y \leq 1$.

8. (6 p.) Are the following series convergent or divergent?

If convergent, is then absolutely or conditionally?

$$\text{a.) } \sum_{n=2}^{\infty} \frac{2}{n(\ln n)^3}, \quad \text{b.) } \sum_{n=1}^{\infty} \frac{(-1)^n 6^n}{1+n6^n}, \quad \text{c.) } \sum_{n=3}^{\infty} \left(\frac{2}{n} - 1\right)^n.$$

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