Name: _____

Calculus

Lin. Alg.

1.	2.	3.	4.	$\sum$	5.	6.	7.	8.	$\sum$
6/	8/	6/	5/	25____	7/	6/	6/	6/	25____

Mathematics II. (BSc)– 1st Midterm Test
4th of April, 2012.

1. **Calculus examples**

   (You need reach at least 8 points to pass this part.)

2. (6 p.) Are the following series convergent or divergent?

$$\text{a.)} \sum_{n=1}^{\infty} \left(\frac{3n+1}{3n+3} \right)^{6n^2}, \qquad \text{b.)} \sum_{n=1}^{\infty} \frac{3^{2n+1}}{5^n \sqrt{n}}.$$

3. (8 p.) a.) Give the domain of the convergence for the series:

$$\sum_{n=1}^{\infty} \frac{(-3)^n}{\sqrt[3]{n}} x^n.$$

   b.) Find the domain of convergence and the sum of the series:

$$\sum_{n=3}^{\infty} \frac{x^n}{n-2}.$$

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4. (6 p.) Find the following limit:

$$\lim_{x \to 0} \sum_{n=1}^{\infty} \frac{(-1)^n \cos(n^2 x + 1)}{3x^2 + 4^n}.$$

5. (5 p.) Find Taylor series at $x_o = 0$ for the function

$$f(x) = \frac{1}{\sqrt[5]{1 + 6x^3}}$$

and give the radius of the convergence. Give the values of $f^{(9)}(0)$ with the elementary operations!

Linear Algebra examples

(You need reach at least 8 points to pass this part.)

5. (7 p.) Find the eigenvectors and eigenvalues of the matrix

$$\underline{\underline{A}} = \begin{pmatrix} 4 & 0 & 2 \\ 0 & 2 & -2 \\ 2 & -2 & 3 \end{pmatrix}.$$

6. (6 p.) a.) $\underline{\underline{A}}^{-1} =?$ if

$$\underline{\underline{A}} = \begin{pmatrix} 1 & 1 & 5 \\ 2 & 4 & 8 \\ -4 & 2 & -9 \end{pmatrix}.$$

b.) $\underline{\underline{X}} =?$ if

$$\begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix} \cdot \underline{\underline{X}} \cdot \begin{pmatrix} -5 & 1 \\ 2 & 3 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}.$$

7. (6 p.) At what values of a has the linear equation system infinitely many solutions? Give the

   solution set in this case!

$$\begin{aligned} 2x + 3y + 5z &= -1 \\ x + 4y + 2z &= 2 \\ 4x + 11y + 9z &= a \end{aligned}$$

8. (6 p.) Which vectors form linearly independent system?
   a.)

   $\underline{a} = \begin{pmatrix} 2 & 0 & 0 & 4 \end{pmatrix}, \underline{b} = \begin{pmatrix} 0 & 6 & 0 & -2 \end{pmatrix}, \underline{c} = \begin{pmatrix} 4 & 0 & 2 & 0 \end{pmatrix}, \underline{d} = \begin{pmatrix} -2 & 6 & 0 & -2 \end{pmatrix}$

   b.)

   $\underline{x} = \begin{pmatrix} 2 & 6 & 0 & 2 \end{pmatrix}, \underline{y} = \begin{pmatrix} 2 & 6 & 2 & -2 \end{pmatrix}, \underline{z} = \begin{pmatrix} 1 & 0 & 0 & 2 \end{pmatrix}, \underline{w} = \begin{pmatrix} 1 & -3 & 0 & 1 \end{pmatrix}$

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