1. (6 p.) Let

$$f(x) = 1 + \sin^{-1}\left(\frac{1+x}{2+x}\right).$$

a.) Find the domain, the range and the derivative of $f(x)$.

b.) Show that the inverse of f exists and find it.

c.) Find the domain, the range and the derivative of the inverse of $f(x)$.

Solution: a.) $x \in [-\frac{3}{2},\infty], f(x) \in [1-\frac{\pi}{2}, 1+\frac{\pi}{2}), f'(x) = \frac{d}{dx}(1+\sin^{-1}(\frac{1+x}{2+x})) = \frac{1}{(4x+x^2+4)\sqrt{\frac{1}{(x+2)^2}(2x+3)}}$

b.) $f'(x) > 0 \Rightarrow f$ strictly monotone increasing $\Rightarrow \exists f^{-1}$

$x = 1+\sin^{-1}\left(\frac{1+y}{2+y}\right) \Rightarrow \sin(x+1) = \frac{1+y}{2+y} \Rightarrow y = \frac{2\sin(x+1)-1}{\sin(x+1)-1}$

c.) $D_{f^{-1}} = R_f, R_{f^{-1}} = D_f, \frac{d}{dx}\left(\frac{2\sin(x+1)-1}{\sin(x+1)-1}\right) = -\frac{\cos(x+1)}{\frac{3}{2}-\frac{1}{2}\cos(2x+2)-2\sin(x+1)}$

2. (3+4 p.) a.) Find an equation for the plane passing through the given point $P(3,1,-1)$ and parallel to the lines

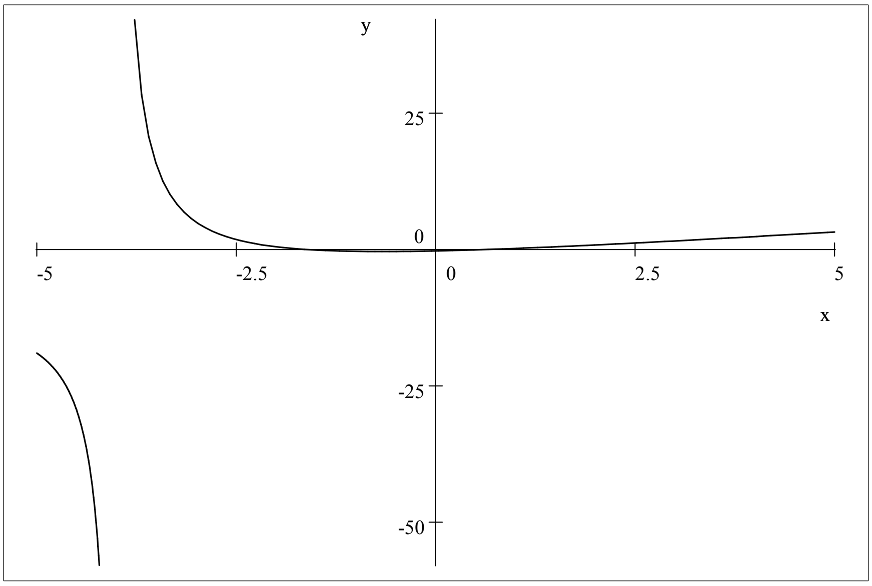
$$e_1 : 2x = -y = 3z \quad \text{and} \quad e_2 : 5(x-7) = -2(y+3) = 10(z-4).$$

b.) Solve the equation and give the result in algebric form:

$$(\sqrt{3}+i)z^4 + 2i = 0.$$

3. (7 p.) Sketch the graph of

$$f(x) = \frac{x^2+x-1}{x+4}.$$



Solution: $x \neq -4, x^2+x-1 = 0$, Solution is: $-\frac{1}{2}\sqrt{5}-\frac{1}{2}, \frac{1}{2}\sqrt{5}-\frac{1}{2}$ (roots)

$f'(x) = \frac{d}{dx}\frac{x^2+x-1}{x+4} = \frac{1}{x+4}(2x+1) + \frac{1}{8x+x^2+16}(1-x^2-x) = 0$, Solution is: $-\sqrt{11}-4, \sqrt{11}-4$ (stationary points)

$f''(x) = \frac{d}{dx}\left(\frac{1}{x+4}(2x+1) + \frac{1}{8x+x^2+16}(1-x^2-x)\right) = \frac{2}{x+4} + 2\frac{-2x-1}{8x+x^2+16} + \frac{2x+2x^2-2}{48x+12x^2+x^3+64} = 0$, No solution found. (No inflection)

$\frac{x^2+x-1}{x+4} = x-3+\frac{10}{x+4} \Rightarrow y = x-3$ oblique asymptote, $x = -4$ vertical asymptote.

4. (6 p.) Let given

$$f(x) = 3\cos\left(x^2+\frac{\pi}{2}\right) \quad \text{and} \quad g(x) = \tan^{-1}\frac{1}{x}.$$

a.) $f \circ g(x) = ?$ and $D_{f\circ g} = ?$ b.) $(f \circ g)'(x) = ?$ c.) $\lim_{x\to\infty} f \circ g(x) = ?$

5. (6 p.) a.) Determine the 2015th derivatives ($f^{(2015)}(x) = ?$) of the following functions:

$$f(x) = (x^2 - 1)e^x.$$

$f^{(2015)}(x) = \frac{d}{dx^{2015}}(x^2 - 1)e^x = 4058\,209e^x + 4030xe^x + x^2e^x$

b.) $\lim_{x \to 1-} (x^2 - 1)e^x = 0$ c.) $\lim_{x \to -\infty} (x^2 - 1)e^x = 0$

6. (7 p.) Evaluate the integrals:

$$\text{a.)} \int \frac{2x - 3}{x^2(x - 1)}dx, \qquad \text{b.)} \int \sqrt{x^2 - 4}\,dx.$$

a.) $\ln x - \frac{3}{x} - \ln(x - 1)$ b.) $\frac{1}{2}x\sqrt{x^2 - 4} - 2\ln\left(x + \sqrt{x^2 - 4}\right)$

7. (6 p.) Evaluate the definite integrals:

$$\text{a.)} \int_1^2 \frac{e^{\sqrt{x}}}{\sqrt{x}}dx, \qquad \text{b.)} \int_{-3}^{-2} \frac{2x + 4}{x^2 + 6x + 10}dx.$$

a.) $2e^{\sqrt{2}} - 2e$ b.)

8. (5 p.) Draw the area between the given curves and calculate the value of the area:

$$y = e^{-2(x-1)}, \quad y = e^{x-1}, \quad y = e^2.$$