Name: _____

1.	2.	3.	4.	5.	6.	7.	8.	Σ
6/	7/	7/	6/	6/	7/	6/	5/	50___

Mathematics I. (BSc)– Exam Test V.
17th of Jun, 2015.

You need reach at least 20 points to pass.

1. (6 p.) Let

$$f(x) = 1 + \sin^{-1}\left(\frac{1+x}{2+x}\right).$$

a.) Find the domain, the range and the derivative of $f(x)$.
b.) Show that the inverse of f exists and find it.
c.) Find the domain, the range and the derivative of the inverse of $f(x)$.

2. (3+4 p.) a.) Find an equation for the plane passing through the given point $P(3, 1, -1)$ and parallel to the lines

$$e_1 : 2x = -y = 3z \quad \text{and} \quad e_2 : 5(x-7) = -2(y+3) = 10(z-4).$$

b.) Solve the equation and give the result in algebric form:

$$(\sqrt{3} + i)z^4 + 2i = 0.$$

3. (7 p.) Sketch the graph of

$$f(x) = \frac{x^2 + x - 1}{x + 4}.$$

4. (6 p.) Let given

$$f(x) = 3\cos\left(x^2 + \frac{\pi}{2}\right) \quad \text{and} \quad g(x) = \tan^{-1}\frac{1}{x}.$$

a.) $f \circ g(x) = ?$ and $D_{f \circ g} = ?$ b.) $(f \circ g)'(x) = ?$ c.) $\lim_{x \to \infty} f \circ g(x) = ?$

5. (6 p.) a.) Determine the 2015th derivatives ($f^{(2015)}(x) = ?$) of the following functions:

$$f(x) = (x^2 - 1)e^x.$$

b.) $\lim_{x \to 1-} f(x) = ?$ c.) $\lim_{x \to -\infty} f(x) = ?$

6. (7 p.) Evaluate the integrals:

a.) $\int \frac{2x-3}{x^2(x-1)} dx,$ b.) $\int \sqrt{x^2 - 4} \, dx.$

7. (6 p.) Evaluate the definite integrals:

a.) $\int_1^2 \frac{e^{\sqrt{x}}}{\sqrt{x}} dx,$ b.) $\int_{-3}^{-2} \frac{2x+4}{x^2 + 6x + 10} dx.$

8. (5 p.) Draw the area between the given curves and calculate the value of the area:

$$y = e^{-2(x-1)}, \quad y = e^{x-1}, \quad y = e^2.$$